Peppermill Lake Nutrient Budget

Prepared for the Peppermill Lake Protection District

By Paul McGinley

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Introduction

The quantity and type of fish, plants and other life in the lake reflect the nutrient input and how that influences the food web. The rate at which a limiting nutrient, such as phosphorus or nitrogen, is made available can determine the growth rate of algae and plants. Other factors, such as light, temperature and sediment composition are also important but are very difficult or impossible to control. The rate of nutrient input can be increased by changes to land that result in increasing or decreasing the quantity of nutrients moving to the lake. Changes to nutrient input cascade through the biology of a lake in complex ways.

An important characteristic of shallow lakes is how changes in phosphorus or nitrogen entering the lake affect the balance between aquatic plants rooted in the sediment and algae growing in the water. In addition to simply competing for phosphorus and nitrogen, plants and algae influence the lake in other ways that can benefit one or the other. For example, aquatic plants provide cover for the microscopic animals (zooplankton) that feed on, and thereby reduce, algae in the water. Similarly, the presence of algae in the water can block sunlight, and thereby slow the growth of aquatic plants. These are examples of some of the positive feedbacks that can lead a shallow lake to be either a clear-water (plant-dominated) or a turbid-water (algae-dominated) lake. The transition from a clear to turbid lake can be abrupt or gradual, and reversal back to a clear, plant-dominated lake can be slow. The controls on this transition from clear to turbid has been a research area for several decades and is still not completely understood. It seems that an important determinant of whether a shallow lake has clear or turbid water is the phosphorus concentration in the lake. Turbid lakes are more likely at higher total phosphorus concentrations as that supports the growth of suspended algae. Early research suggested this transition could be abrupt in response to an increase in phosphorus and that once a lake “flips” from a clear-water to turbid-water state, it is not easy to reverse back to a clear-water state. Recent research is also suggesting, however, that transitions in shallow lakes can occur more slowly through several steps of lake change that accompany increasing phosphorus concentrations. For example, in a group of shallow lakes in Denmark, lakes with phosphorus concentrations above 40 – 50 ug/l were more likely to exhibit late summer algal blooms and seemed to be changing in ways that would eventually lead to more turbid conditions (Sayer et al., 2010). They found the lakes with higher phosphorus had plant communities that had shorter growing seasons (e.g., more pondweeds (*Potamogeton*) and fewer charophytes). That suggests that over longer periods of time, lakes with intermediate, but increasing nutrient levels may progress to plant communities that that have a shorter plant-covered period opening up a greater possibility for later season algal blooms. These lakes were described as “crashing” as they could be moving to less stable plant communities increasing the likelihood of algal blooms. Another indication of lake change is a proliferation of filamentous algae. Filamentous algae attached to plants can reduce plant growth and thereby move a shallow lake towards more suspended algae in the water (Jones and Sayer, 2003; Iacarella et al., 2018). This filamentous and attached algae can be a response to increased nutrients. A related observation of interest to central Wisconsin lakes is a study that found filamentous algae can also be increased by increased nitrogen concentrations (Olson et al., 2015). The algal growth reportedly led to a decline in aquatic plants and an increase in suspended algae.

It is not easy to identify this progression from clear-water to turbid-water if it occurs with slow changes to aquatic plant communities but managing the entry of nutrients to prevent increases in lake phosphorus and nitrogen concentrations is one of the most important lake management opportunities.

A nutrient budget is a tool for understanding the nutrient loading and opportunities for management. The nutrient budget estimates the input and output of nutrients such as phosphorus, nitrogen and can be used to explore how those might be influencing the condition of the lake. In some cases, these can be measured but in other cases they must be estimated. The result is a simplified representation of the lake or a model of how nutrients enter, are used within and leave the lake.

Figure 1 is a general conceptual model for water and nutrients in Peppermill Lake. To develop a nutrient budget, the annual inputs and outputs of both nitrogen and phosphorus were estimated. This study used a variety of methods to estimate these including a 2002 report on Peppermill Lake prepared by the Center for Watershed Science and Education at UW-Stevens Point, a literature review of nutrient budgets in other Wisconsin lakes, and measurements collected by the Peppermill Lake Protection District (WDNR SWIMS; Evans, 2021).

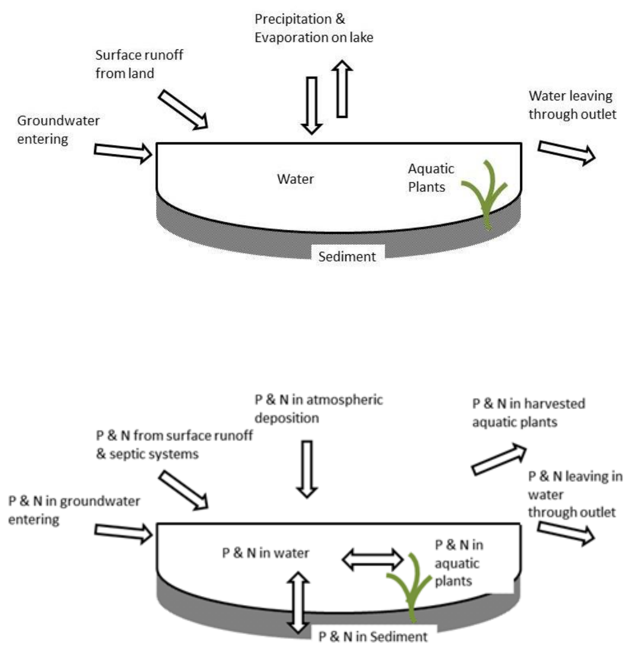

Figure 1. Schematics of the hydrologic (upper) and nutrient (lower) budgets for Peppermill Lake.

The input/output estimates were used to develop a concentration model that was calibrated with concentration measurements made throughout the year by the Peppermill Lake Protection District and archived in the Wisconsin DNR SWIMS databased. The concentration model provides a tool to better understand the lake and how it responds to external and internal nutrient loading. The concentration model incorporates reactions for the loss and gain of a nutrient in the lake. These are simplified representations of the complex processes in the lake designed to fit the concentration data collected over time and provide some insight into how the nutrient is processed in the lake. The model for each nutrient is presented here with a brief overview of how the model was applied. The concentration model is also described in more detail in the Appendix.

Hydrologic Budget

Flow out of the lake was measured three times in 2021/2022 by staff from the Center for Watershed Science & Education at UW-Stevens Point. The flow reflects the rate at which the water is entering the lake and loss from the lake through evaporation.

Table 1. Flow measured downstream of the Peppermill Lake Dam

Date	Flow (cubic feet per second)
9/16/2021	6.6
1/26/2022	5.9
7/18/2022	4.6

Because Peppermill Lake does not have a perennial stream entering the lake and the difference between precipitation and evaporation will be relatively small (less than 0.1 cubic feet per second), the average measured flow of 5.7 cubic feet per second is a reasonable estimate of the groundwater inflow to the lake. Results from a groundwater flow model were used to estimate the land area that is contributing groundwater to Peppermill Lake. Figure 2 shows the land area that is modeled as contributing groundwater to the lake. The full area is approximately 5.2 square miles. To generate an average flow of 5.7 cfs from that area, an annual groundwater recharge of approximately 15 inches would be required. That is several inches per year higher than the average assumed in regional groundwater flow modeling (e.g., Kraft and Mechenich, 2010), but could reflect the recent higher than average precipitation and also reflect some uncertainty in the modeling.

The flow out of the lake in 2020/2021 was higher than it was in 2001 when the mean of measurements made April-September was 3.0 cfs (range of 2.3-3.9 cfs). The higher flow in the more recent 2020/21 measurements is consistent with the period of higher precipitation in 2018-2020. Figure 3 shows the annual precipitation recorded in nearby Montello, Wisconsin. This recent period of higher precipitation has led to higher streamflows in other parts of central Wisconsin as Figure 3 also shows how the streamflow measured in Tenmile creek near Nekoosa was higher in 2020/21 than it was in 2001.

Groundwater can take decades to travel in the aquifer from portions of this groundwater contributing area to the lake. Because the use of nitrogen fertilizers has been higher in the last fifty to sixty years, the portion of the groundwater contributing area that is within fifty years of the lake was also identified. That area is about 3.7 square miles or about 72% of the total groundwater contributing area.

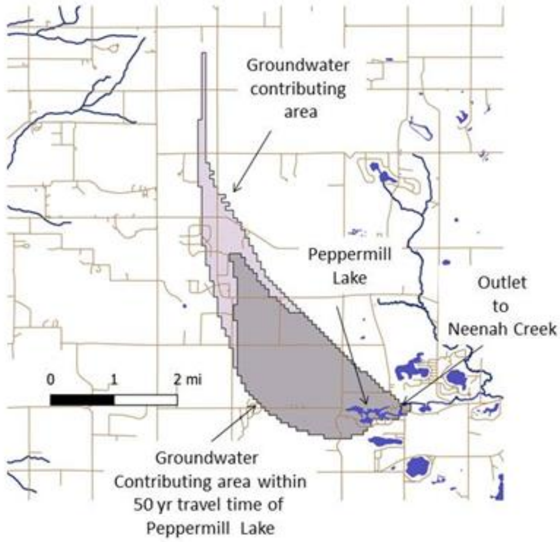

Figure 2. Peppermill Lake groundwater contributing area and that portion that is within 50 year travel time of the lake determined using a groundwater flow model.

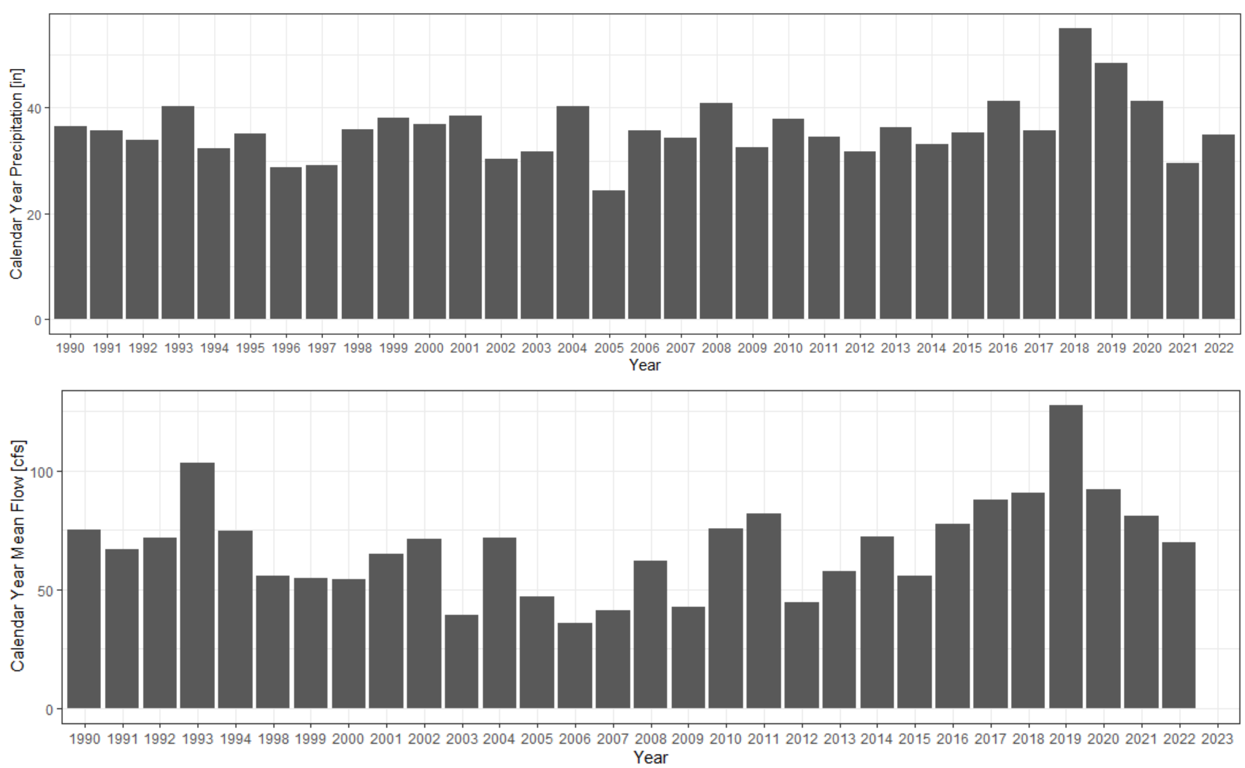

Figure 3. Precipitation and streamflow near Peppermill Lake. Upper figure shows the annual precipitation measured in Montello (Montello Wastewater Treatment Plant) and lower figure shows the average flow in Tenmile Creek at Nekoosa (precipitation from the National Climatic Data Center and flow from the U.S. Geological Survey).

The volume of water in Peppermill Lake was estimated to be 280 acre-feet. With an average flow leaving the lake of 5.7 cfs, there is a 25 day average water residence time in the lake. This is an average residence time and the shape of the lake and the presence of several basins will result in some water spending less and some more time in the lake.

Nitrogen

Nitrogen is an essential element that occurs in many forms in the lake. Nitrogen in the lake can be present as nitrate (NO_3^-) and nitrite (NO_2^-), as ammonia (NH_3/NH_4^+) and as organic nitrogen, along with nitrogen in gaseous forms (primarily nitrogen gas N_2 and some nitrous oxide N_2O). The 2021/22 measurements focused on following nitrate/nitrite concentrations as that is the major form of nitrogen entering the lake. We will refer to these measurements as just nitrate because the nitrite is likely just a very small portion of the total and the analysis reports the total of nitrate plus nitrite. Nitrate is a form of nitrogen that moves readily through soils and groundwater. Once in the lake, it appears to be consumed rapidly in spring and summer. This consumption can be from assimilation into plants and bacteria. It can also be from consumption by bacteria during respiration under low oxygen conditions when it is converted to nitrogen gas (and then lost from the lake) in a process called denitrification. The measured nitrate concentrations in Peppermill Lake showed an increase during the winter and a rapid drop in the spring. That is consistent with a high rate of nitrate consumption during periods of the year with more biological growth (more sunlight, warmer temperatures) and shows the importance of collecting winter nitrate measurements. We were able to simulate this pattern in the concentration model with a rate of disappearance linked to the intensity of sunlight on the lake. Figure 4 shows how this model can fit the observed concentration pattern. The model was fit with an average nitrate concentration entering the lake of 2.3 mg/l or approximately 26,000 pounds/year. That is consistent with an estimated average annual nitrate loading of 25,000 pounds/year from agriculture (i.e., loss of ~60 lb/acre-year from the agricultural land within 50 year travel time) and 790 pounds/year from septic systems (79 parcels at 10 lb/system-yr).

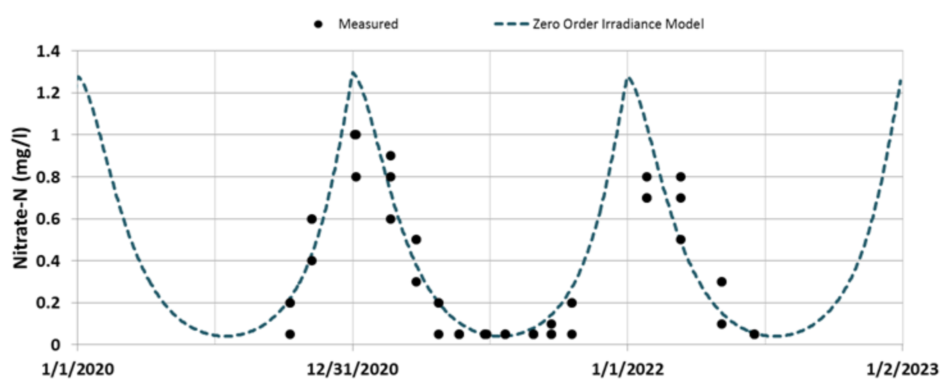

Figure 4. Measured nitrate-nitrogen concentration in Peppermill Lake compared with results of a simulation model that links removal of nitrate with light intensity. The simulation model used an inflow concentration of 2.3 mg N/l and fitted removal rates leading to a summer depletion rate of 140 mg N/m2-day.

Assuming the concentration in the lake is similar to that in the streamflow leaving the lake, only about 2,000 pounds/year of nitrogen leaves the lake as nitrate. The result is approximately 18,000 pounds of nitrate-nitrogen is consumed in the lake each year. The fate of that is likely some mixture of assimilation in plants and algae combined with denitrification.

It is difficult to estimate the amount of nitrogen stored in plants in the lake but assuming an approximate plant density of 50 grams dry plant material/m2 (that would likely be on the low side for chara or milfoil stands but perhaps reasonable as a lake-wide average based on an earlier study in nearby McGinnis Lake), leads to a lake-wide aquatic plant dry mass of approximately 25,000 pounds. If the average nitrogen content is 2%, there would be 500 pounds of nitrogen within the plants in the lake. Assuming the plant harvesting removes approximately 150,000 pounds of wet plants annually (300 yd3) and assuming they are 92% moisture and 2% nitrogen on a dry mass basis, the plant harvest would be removing approximately 240 pounds of nitrogen each year.

Based on measurements made in 2001, the sum of the organic and ammonia nitrogen concentration in the lake is approximately 0.55 mg/l. That would lead to approximately 6,000 pounds of nitrogen leaving the lake as organic and ammonia nitrogen. This nitrogen is largely generated within the lake through conversion of nitrogen in the sediment or incoming groundwater into plant and algal biomass that subsequently decomposes and then leaves the lake through the outflow.

The estimates of annual inflow and outflow of nitrate-nitrogen is combined with estimates of the quantity of nitrogen in the lake is summarized in Figure 5.

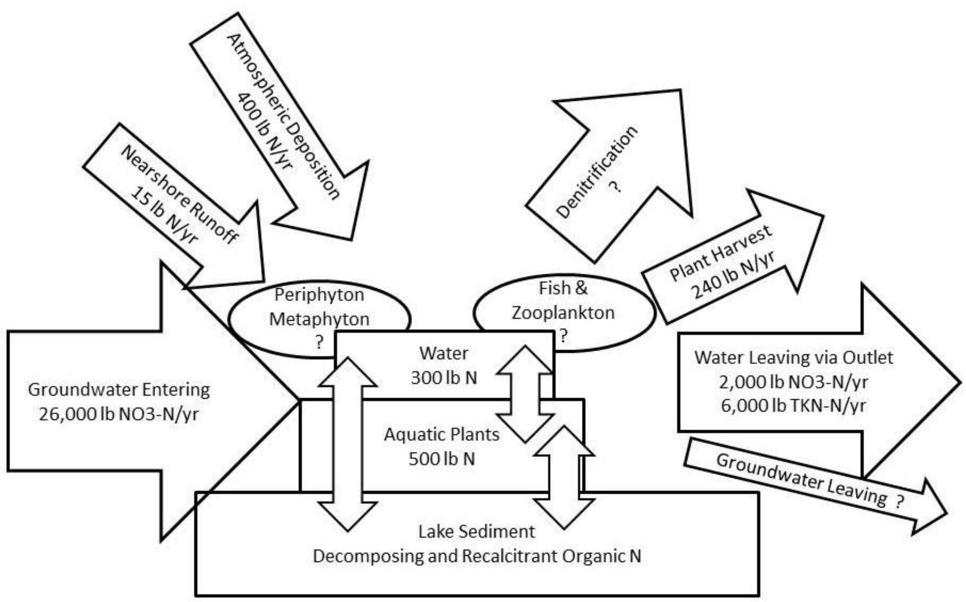

Figure 5. Schematic summary of the annual nitrogen inputs, outputs and in-lake components of the nitrogen budget in Peppermill Lake. In-lake components based on mid-summer estimates of total nitrogen concentration and plant density.

While the in-lake concentration and plant biomass are a small percentage of the annual input and output of nitrogen, they represent a measure of the role of nitrogen in the biological productivity in the lake and are an available source of available nitrogen when they decompose. Aquatic plant harvesting appears to remove a significant fraction of the of the aquatic plant biomass that would otherwise be subject to decomposition and cycling of nitrogen.

It is important to understand the fate of the large nitrogen loading from groundwater. Rooted aquatic plants obtain most of their nitrogen and phosphorus from the sediments. Recent research (for example, Sondergaard et al., 2017) indicates that the combination of both nitrogen and phosphorus can influence shallow lake productivity. At moderately high phosphorus concentrations, increasing nitrogen loading has been shown to reduce plant coverage and lead to a more turbid lake where suspended algae reduce light transmission and discourage the growth of aquatic plants. Other research has shown that increased nitrogen loads can increase the quantity of filamentous algae attached to plants and on the lake bottom (Yang et al., 2020; Trochine et al. 2010). While the impact that the groundwater nitrogen loading is having on Peppermill Lake is not known, the nutrient budget suggests reducing external loads of nitrogen to the lake is a useful management strategy.

Phosphorus

The largest external sources of phosphorus to Peppermill Lake are groundwater, atmospheric deposition (phosphorus in dust, pollen, precipitation etc) and runoff from the near-shore area. These were estimated using measurements made in 2002 and using information from other studies in Wisconsin. The quantity of phosphorus entering in groundwater was estimated as the product of the average flow of 5.7 cfs and a concentration of 0.0075 mg/l. As described earlier, the average flow out of the lake is considered a reasonable estimate of the groundwater flow into the lake. The concentration in groundwater was based on water samples collected from mini-piezometers in the lakebed in 2002 that seemed representative of water not stored in the lake bed or drawn from the lake (i.e., low ammonium, high velocity and measureable nitrate). With those values, it was estimated that 84 pounds per year of phosphorus enter the lake each year in groundwater.

Atmospheric deposition was estimated at 0.3 lb/acre-yr from previous studies in Wisconsin for small lakes leading to a total of 15 pounds per year of phosphorus entering the lake.

Surface runoff into the lake was estimated by assuming there would be several inches of runoff each year directed into the lake from the land area within 100 feet of the water. This was a very approximate way to estimate the magnitude and relative importance of this part of the nutrient budget. Using GIS, this area was estimated to be 44 acres. For two inches of runoff per year at a concentration of 1.0 mg P/l (based on previous studies on runoff from lakeshore lots such as Graczyk et al., 2002), approximately 20 pounds of phosphorus per year would enter the lake.

The septic system contribution was estimated as 0.1 pounds per person per year assuming that 10% of the phosphorus released from each septic system would actually make it to the lake after reactions in the soil. This is also a very general estimate that provides a way to incorporate this term in the budget and will benefit from additional research. We assumed 100 people-years of septic usage based on the number of parcels in the groundwater contributing area. That would lead to 10 pounds per year of phosphorus entering the lake from septic systems.

The phosphorus from groundwater, atmospheric deposition, runoff and septic systems combined was estimated to be 129 pounds per year. In contrast, the amount of phosphorus estimated to be leaving the lake in the outflow was 232 pounds/year and the amount of phosphorus in harvested plants was another 24 pounds. This difference between inflow and outflow phosphorus mass was noted in the 2002 study and is attributed to recycling of phosphorus that is already present in the lake sediment. This includes phosphorus that mixes or diffuses out of the sediment or is first taken up by aquatic plants and attached algae and is released when the plants decompose (Barko and Smart, 1980; Jensen et al., 2006; Li et al., 2021). As a result, the measured concentration of phosphorus in the lake can vary depending on the rate of this release. Figure 5 shows that the measured concentrations vary widely but seem likely to be higher during the summer. The model assumes a release of phosphorus that increases as the temperature warms and decreases as it cools consistent with decomposition of organic matter and resuspension during ice-free periods. While the model describes a trend of increasing phosphorus during the summer, it does not capture the wide variability in phosphorus concentrations that can be observed. The measured results are consistent with a much more episodic transfer of phosphorus from the sediment and decomposing plants and algae into the water than is simulated in a model based only on temperature.

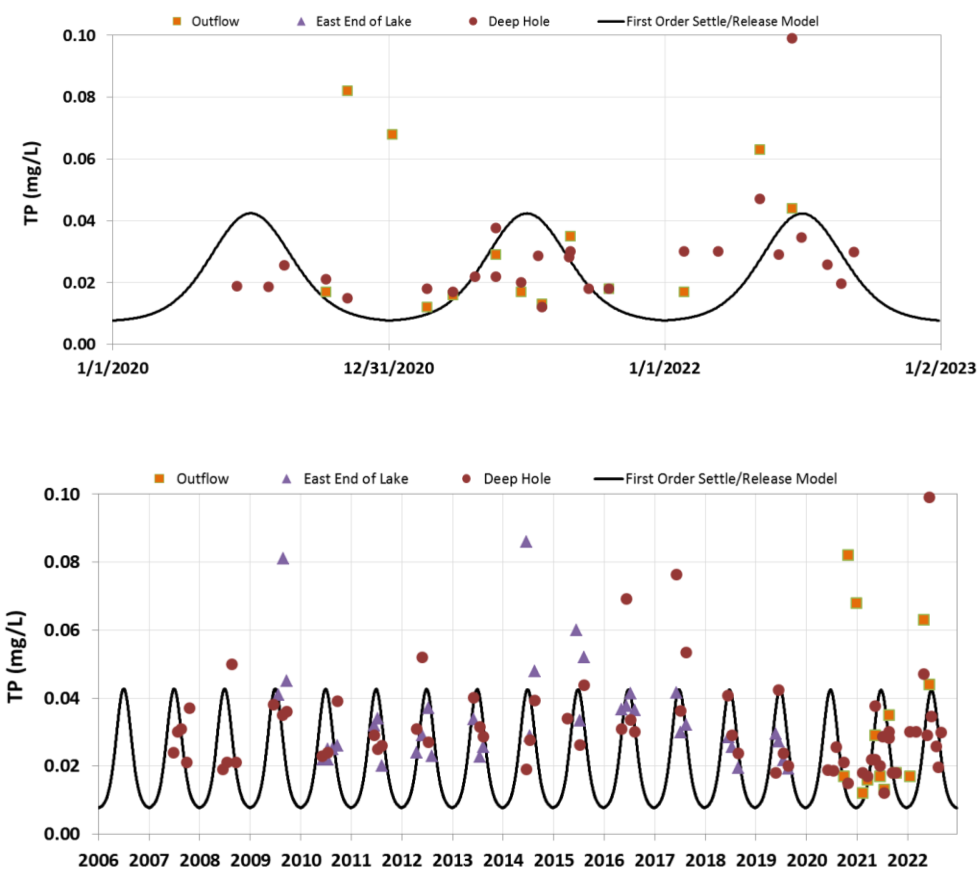

Figure 5. Monitoring results of total phosphorus concentrations compared to results from a simulation model combining settling from the water and release from the sediment. Model simulation assumes external phosphorus loading was 129 lb/yr and the net internal addition of phosphorus was 102 lb/yr.

In most Wisconsin lakes, the in-lake phosphorus concentration is correlated with the suspended algal in the lake. This is exhibited in Peppermill Lake by the relationship between increasing chlorophyll a with increasing phosphorus concentration shown in Figure 6. Although there is considerable scatter in the relationship, higher phosphorus concentrations is frequently associated with higher chlorophyll concentrations. This likely reflects both the incorporation of phosphorus within algae, so that measurement of phosphorus includes the phosphorus in algae, and that increasing phosphorus can lead to more algae in the lake.

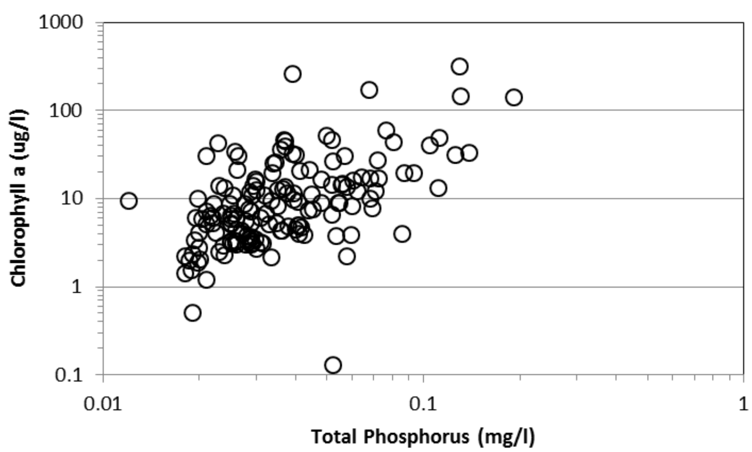

Figure 6. Relationship between measured total phosphorus and chlorophyll a in Peppermill Lake (all seasons and all locations) showing how higher phosphorus concentration are associated with higher levels of measured chlorophyll a.

Figure 7 is a conceptual summary of the phosphorus fluxes into and out of the lake along with some estimates of the in-lake storage of phosphorus in the water and aquatic plants. As discussed above, this shows Peppermill Lake as a source of phosphorus with more leaving than entering. This can be explained by aquatic plants and algae obtaining phosphorus from the sediment in the lake, which leads to plant and algal growth and subsequent release to the water as they decompose. The large outflow from the lake then conveys a portion of this phosphorus out of the lake. This conceptual model requires that the sediment provide a reservoir of phosphorus that is accessed by aquatic plants and algae. Based on measurements made in other lakes and the history of Peppermill Lake as an impoundment created within a groundwater-fed wetland, it seems reasonable that there is a reservoir of phosphorus available within the substrate. Rooted aquatic plants and algae can acquire this phosphorus and move it into biomass that later decomposes and releases phosphorus.

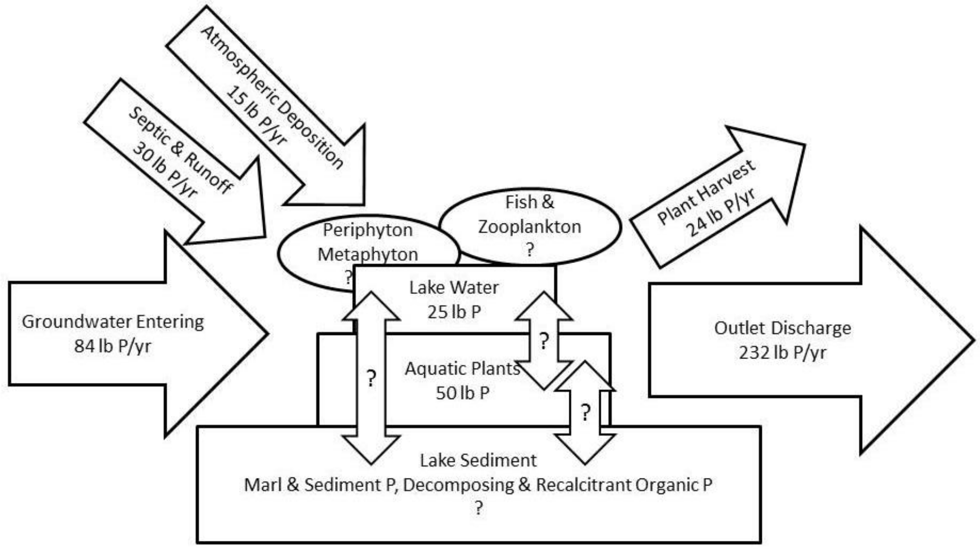

Figure 4. Schematic summary of the annual phosphorus inputs, outputs and in-lake components of the phosphorus budget in Peppermill Lake. In-lake components based on mid-summer estimates of total phosphorus concentration and plant density.

Management Implications & Recommendations

Peppermill Lake is a shallow, hardwater lake fed by groundwater with productive aquatic plant and algal communities. These communities use nitrogen and phosphorus which can be released during decomposition and recycled within the lake. This internal recycling leads to summer total phosphorus concentrations that can support suspended algal communities. Important questions remain about how the nutrient loading is influencing the balance between aquatic plants and algae.

Peppermill Lake receives large quantities of nitrate-nitrogen in groundwater. This is primarily from agricultural land in the watershed (~90%) and septic systems (~5%). Most of this nitrate-nitrogen is consumed within the lake although how this consumption influences the aquatic plants and algae is not known. If this additional nitrogen is increasing the rate of biological productivity in the lake or increasing the rate of organic matter decomposition, it may also lead to an increase in available nitrogen and phosphorus. Reducing the amount of nitrogen that is entering the lake in groundwater will require farmers in the watershed find ways to reduce the amount of nitrogen that is lost to groundwater.

Peppermill Lake also receives external phosphorus from near-shore runoff and septic systems. This is one of the few controllable sources of phosphorus. It would benefit the lake to reduce near-shore runoff by infiltrating rainwater and making sure that runoff from impervious surfaces is directed to infiltration areas far from the lake.

The biology of Peppermill Lake is dominated by aquatic plants and rooted algae (i.e., *Chara*). These plants use nutrients from the sediment and the water but once they decompose, these nutrients are released to the water. Aquatic plant harvesting can be important to users of shallow lakes as it opens navigation paths and improves recreational use. Shallow lakes benefit from the presence of aquatic plants because the plants use nutrients that could be used by algae, provide refuge for zooplankton that feed on algae, and prevent suspension of sediments that make the lake more turbid. Plant harvest should be managed to ensure that the lake is retaining a diverse and healthy aquatic plant community. The nutrient budget shows that removal of harvested aquatic plants also removes nitrogen and phosphorus from the lake. As the calculations in this report show, the amount removed can be a relatively small amount of the overall nutrient budget but nonetheless, it can be removing nutrients that would likely become part of more active fraction of nutrients in the lake.

Future monitoring could include extending the current monitoring to improve the ability to observe changes to the balance between plants and algae. Some suggestions for future monitoring include:

1) Continue the Secchi depth, total phosphorus and chlorophyll measurements during the summer as these are useful ways to track changes to the suspended algal population;
2) Consider adding nitrogen to the routine sampling such as sampling the outflow in the winter and the summer testing locations for nitrate/nitrite, total Kjeldahl and ammonia;
3) Continue to track the plant communities to identify if there are changes in the charophyte, pondweed and invasive populations;
4) Develop a monitoring program to track the density of attached algae and filamentous algae clusters.

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Appendix

Concentration Model for Peppermill Lake

The concentration model considers the lake as a simple mixed system where the change in mass with time in the lake is equal to the change in concentration with time multiplied by the lake volume. This change in mass is also assumed to be equal to the difference between what enters and what leaves the lake each day.

$$dM/dt = (dVC/dt) = Qin*Cin + Qout*Cout - REM + REL$$

where:
M: mass of solute in lake (g)
t: time (d)
V: volume of water in the lake (m3)
C: concentration (g/m3 = mg/l)
Qin: water flow in to the lake (m3/d)
Cin: concentration entering the lake (g/m3)
Qout: water flow leaving the lake (m3/d)
Cout: concentration leaving the lake (g/m3)
REM: reactions within the lake that remove the solute to the water (g/day)
REL: reactions within the lake that add the solute to the water (g/day)

We can simplify this model by assuming the lake volume does not change with time which is appropriate for lakes with relatively high inflow and outflow rates and a regional hydrology with evaporation on the lake approximately equal to precipitation. Here, we also neglect concentration of solutes in the remaining lake water when ice forms which is a reasonable approximation if the ice volume is a small percentage of the total lake volume. This simplifies the water budget so that we can assume the flow in (Qin) is equal to the flow out (Qout).

$$V\, dC/dt = Q\,(Cin-Cout) - REM + REL$$

Assuming that the lake is mixed allows us to assume the concentration leaving the lake (Cout) is the same as the concentration in the lake (CL). Dividing both sides by the volume of the lake results in:

$$dC/dt = (Q/V)\,(Cin-CL) - REM/V + REL/V$$

Following the approach used by Jensen et al. (2006) we can multiply both sides by the mean depth of the lake, Z:

$$Z*\, dC/dt = (Q/V) * (Z*Cin - Z*CL) - (Z/V)\, REM + (Z/V)\, REL$$

Which when combined, Z*C becomes grams/m2 or is expressing the concentrations as mass of solute per lake area and the removal (REM) and release (REL) terms are now expressed as:

$$REM' = (Z/V)*\, REM$$
Where REM' is in grams/m2-d

REL' = (Z/V) REL
Where REL' is in grams/m2-d

We can rewrite our expression for the concentration of different solutes now using a letter for the solute when the concentration is expressed as a mass per area (similar to Jensen et al., 2006):

dP/dt = (Q/V) * (Pin-PL) – REM' +REL'

where:
P is the phosphorus concentration in gram/m2
Q is the flow in and out (m3/d)
V is the volume (m3)
Pin is the phosphorus concentration in the water entering (in g/m2, which is calculated as the mass entering the lake in a day divided by the volume entering that day and by the area of the lake)
PL is the phosphorus concentration in the lake expressed as g/m2 (calculated as the mass of P in the lake divided by the area or the concentration of P in the lake in grams/m3 multiplied by the depth Z in meters.

For phosphorus, the REM' term as described by Jensen et al. (2006) was used:

REM' = bS * $(1+tS)^{(T-20)}$ * PL/Z
Where
bS: is the settling (or removal in general) velocity (m/d)
tS: is a temperature adjustment which if 0.072 leads to an approximate doubling of the rate for each 10 degree C and is sometimes called the Q10 model (e.g., see Heinz and Stefan, 2003).

In the phosphorus model for Peppermill Lake, the settling velocity was assumed independent of temperature (tS = 0) and we used a settling velocity was 20 m/year. This is higher than the 10 m/year suggested by Vollenweider as expected because their model does not separately include the release rate.

The REL' term has a similar form but is a function of a sediment phosphorus PS rather than the lake concentration

REM' = bF * $(1+tF)^{(T-20)}$ * Ps

Where bF is a first-order release rate constant (1/day), tF is the temperature adjustment and Ps is the sediment phosphorus concentration (g/m2). This requires tracking the sediment phosphorus concentration as it gains phosphorus from sedimentation and loses it from release. Following Jensen et al. (2006), a typical sediment phosphorus is 40 to 60 g/m2. While that is an adjustable parameter in their model, if we assume a sediment P of 300 g/m2, at 20 degree C, the parameterization to obtain perhaps a typical release rate of 10 mg/m2-day = 0.01 g/m2-day:

0.01 g/m2-d = bF * 1 * 300 g/m2

bf = 0.000033 /day

In our fitted model, we used a bf=0.00008 which would correspond to 24 mg/m2-d at 20 degree C.